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INFRAMARGINAL MODELS OF SPATIALLY ALLOCATED ECONOMIC STRUCTURES AND THE ANALYSIS OF PRODUCTION PROCESSES

The article discusses designing of labor division networks. Designing of the economic structure of labor division constitutes the main part of inframaginal analysis. Inframaginal analysis normally uses predefined economic structures, which means that in certain cases some economic structures may be neglected. Such inaccuracies may be not important in the analysis of small enterprises but in the analysis of spatially allocated economic structures, some important aspects may be left unnoticed, which will lead to wrong decisions regarding labor allocation.

To make an enterprise competitive it is essential to understand what is the optimal economic organization and the form of labor division in the given region. If some economic structures are not taken into account in the analysis, the general equilibrium will be incorrect, which will negatively affect the decision-making.

If we use inframarginal models to analyze the production process, it will allow us to take a fresh perspective on the problem. All possible structures of the division of labor are designed by using production factors and goods to reduce the risk of errors in the process of decision-making, which will make the production process of the enterprise more efficient.

Keywords: inframarginal analysis, technology, division of labor, network effects, economic structures, regional economy.

Introduction

There are two types of business decisions: decisions associated with the choice of activity and decisions of resource allocation. Decisions of the first type can be illustrated by the choice of majors students make when entering the university. These are inframarginal decisions. Then students choose the courses they want to study and decide on the time they want to spend on each of the learning courses. These are decisions of the second type – marginal decisions of time allocation. In the context of the division of labor, inframarginal decisions are more important than marginal decisions.

In most cases of inframarginal analysis, a set of economic activities which can be chosen by individuals is set exogenously and inframarginalists are concerned with the problem of mathematical optimization of utility functions [4,14]. The set of economic activities which can be used in the division of labor is usually limited and well known. In real life, however, managers have enough practical experience to determine the optimality of particular structures of the division of labor in various cases. Complex and specific results of inframarginal articles are not practically useful for the decision-making process, which leads to a situation when 'inframarginalists write papers mainly for inframarginalists' [6, p.177].

The technology-oriented theory of production can be divided into f*unction analysis* and a*ctivity analysis* depending on the object of analysis [1, p.1055]. Inframarginal analysis is based on activity analysis, proposed by Koopmans. Function analysis was introduced by Fandel [7, p. 41] to find the types of possible economic structures in the process of inframarginal analysis. '*Activity'* was defined by Koopmans as 'the combination of certain qualitatively defined commodities in fixed quantitative ratios as "inputs" to produce as "outputs" certain other commodities in fixed quantitative ratios to the inputs' [9].

Method and model

Let us now consider the asymmetric model with trading activities and heterogeneous parameters introduced by Yang [13, p.111]. In the model of specialization, there are three types of goods *x,y,*and *z*. The number of goods which are sold on the market have index *s*. The number of goods which are purchased on the market have index *d*. The self-provided goods have index *c.* The transaction cost coefficient is $1-k$, *k* is viewed as a transaction service and depends on the quantity of labor used in transactions. As a service, it can be self-provided or purchased on the market:

$$k = r_c + r_d$$

In this case, r_c and r_d as transaction services relate to the distance between a pair of trade partners and their location problems. All individuals are evenly spaced and the distance between each pair of neighbors is a constant. The distance between a pair of trade partners may differ from the distance between a pair of neighbors. For example, they can be engaged in rural or urban relations.

The utility function is identical for all individuals and has a form of the Cobb-Douglas utility function [5, p.337]:

$$u = [x_c + (r_c + r_d)x_d]^{\alpha}[y_c + (r_c + r_d)y_d]^{\beta}[z_c + (r_c + r_d)z_d]^{\gamma}$$

The set of activities known to an enterprise describes the technical opportunities of this enterprise. This set is called ‘technology’ and is designated by symbol ‘*T*’ [7, p.43].

Therefore, technology can be written the following way:

$$T = \left\{ \begin{pmatrix} -l \\ x \\ y \\ z \\ r \end{pmatrix} \middle| \, l = 1, x, y, z \geq 0 \right\}$$

Labor restrictions are equal for all economic agents and can be written as:

$$l_x + l_y + l_z + l_r = 1,$$
$$l_i \in [0,1],$$
$$i = x, y, z, r.$$

Using the theorem of optimum configuration ‘the optimum decision does not involve selling more than one good, does not involve selling and buying the same good, and does not involve buying and producing the same good’ [11], we can find vectors of activities for technology *T*.

The producer-consumer uses only one production factor *l (labor)* in the production processes. The economic agent can produce a good only for their own consumption x_c or produce an additional part of the good for sale in order to purchase other types of goods that the economic agent does not produce on their own x_s. If the economic agent does not produce a good and purchases the good on the market, we put 0 (zero) in the activity vector.

All the possible activities vectors can be written the following way:

$$T^{'} = \left\{\begin{array}{c}
\begin{pmatrix}-l\\x_c\\y_c\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c\\z_c\\0\end{pmatrix}\\
\begin{pmatrix}-l\\x_c+x_s\\0\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c+x_s\\y_c\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c+x_s\\y_c\\z_c\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c+y_s\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c+y_s\\z_c\\0\end{pmatrix},\\
\begin{pmatrix}-l\\0\\y_c\\z_c+z_s\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\0\\z_c+z_s\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c\\z_c+z_s\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c\\z_c\\r_c+r_s\end{pmatrix},\begin{pmatrix}-l\\x_c\\0\\z_c\\r_c+r_s\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c\\0\\r_c+r_s\end{pmatrix},\\
\begin{pmatrix}-l\\x_c+x_s\\0\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c+x_s\\y_c\\0\\0\end{pmatrix},\begin{pmatrix}-l\\x_c+x_s\\0\\z_c\\0\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c+y_s\\0\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\0\\r_c\end{pmatrix},\\
\begin{pmatrix}-l\\x_c\\0\\z_c+z_s\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c\\z_c+z_s\\0\end{pmatrix},\begin{pmatrix}-l\\0\\0\\z_c+z_s\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\0\\0\\r_c+r_s\end{pmatrix},\begin{pmatrix}-l\\0\\y_c\\0\\r_c+r_s\end{pmatrix},\begin{pmatrix}-l\\0\\0\\z_c\\r_c+r_s\end{pmatrix},\\
\begin{pmatrix}-l\\x_c+x_s\\0\\0\\0\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\0\\0\end{pmatrix},\begin{pmatrix}-l\\0\\0\\z_c+z_s\\0\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c\\0\\r_c+r_s\end{pmatrix}
\end{array}\right\}$$

Each element in matrix *T* represents the production function of an economic agent. The economic agent can choose any production function. The agent's choice represents their production process, and it will be an inframarginal choice. For each activities vector in matrix *T* we will find cases in which the utility functions of the economic agent will be positive. Combinations of activities vectors will give different types of economic structures.

Some of these were reviewed earlier [13, p. 115] and we will use them to show the method of construction of economic structures from technology matrix *T*.

Results

The simplest case is 'autarky': an individual self-provides three goods. Therefore, the number of goods sold and purchased and the number of transaction services are 0. The technology has only one activity vector $T = \left\{\begin{pmatrix}-l\\x_c\\y_c\\z_c\\0\end{pmatrix}\right\}$. The utility function can be written as:

$$u = {x_c}^{\alpha}{y_c}^{\beta}{z^{\gamma}}_c > 0$$

In this case only one activity vector is sufficient to achieve a positive value of the utility function and there is no network of labor division. The pattern of labor division is shown as a graph [2] in Figure 1.

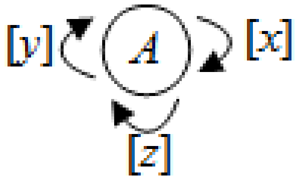

Fig. 1. Autarky.

Activities $\left\{\begin{pmatrix}-l\\x_c+x_s\\0\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c+x_s\\y_c\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\y_c+y_s\\0\\r_c\end{pmatrix}\begin{pmatrix}-l\\0\\y_c\\z_c+z_s\\r_c\end{pmatrix},\begin{pmatrix}-l\\x_c\\0\\z_c+z_s\\r_c\end{pmatrix}\right\}$ exist in cases of partial division of labor. In this case an individual sells one of the produced goods and purchases one of the goods for consumption. The utility function should be positive. For an individual with the activity vector $\begin{pmatrix}-l\\x_c+x_s\\0\\z_c\\r_c\end{pmatrix}$ there should exist an individual with the activity vector $\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\r_c\end{pmatrix}$ and so on for activities $\begin{pmatrix}-l\\x_c+x_s\\y_c\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\r_c\end{pmatrix}$. Individuals form an economic structure with production processes described by technology $T=\left\{\begin{pmatrix}-l\\x_c+x_s\\0\\z_c\\r_c\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\z_c\\r_c\end{pmatrix}\right\}$ (see Figure 2).

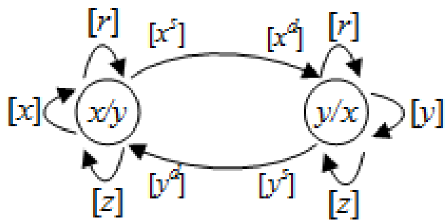

Fig. 2. Partial division of labor.

The complete division of labor (Figure 3) is represented by technology $T=\left\{\begin{pmatrix}-l\\x_c+x_s\\0\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\0\\y_c+y_s\\0\\r_c\end{pmatrix},\begin{pmatrix}-l\\0\\0\\z_c+z_s\\r_c\end{pmatrix}\right\}$ with three activities vectors. In this case, individuals produce only one of the goods and purchase two on the market. The transaction service is self- provided.

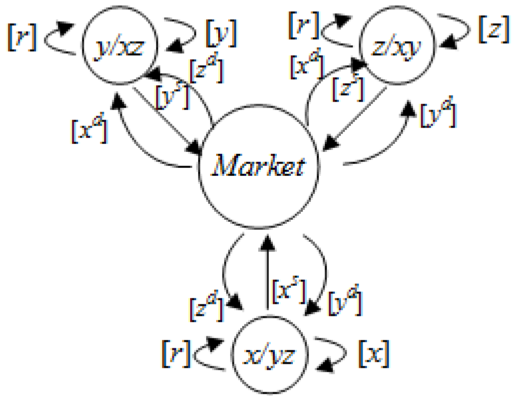

Fig. 3. Complete division of labor.

For a complete production process, an individual with the activity vector $\begin{pmatrix} -l \\ x_c+x_s \\ 0 \\ 0 \\ r_c \end{pmatrix}$ needs two other activities: $\begin{pmatrix} -l \\ 0 \\ y_c+y_s \\ 0 \\ r_c \end{pmatrix}$ and $\begin{pmatrix} -l \\ 0 \\ 0 \\ z_c+z_s \\ r_c \end{pmatrix}$. If all of these activities vectors are present, the utility functions for all individuals are positive and there is division of labor.

Partial division of labor and transaction services can be represented by the combination of the following activities vectors:

$$T = \left\{ \begin{pmatrix} -l \\ x_c + x_s \\ 0 \\ z_c \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c + y_s \\ z_c \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ 0 \\ z_c \\ r_c + r_s \end{pmatrix} \right\}$$

$$T = \left\{ \begin{pmatrix} -l \\ x_c + x_s \\ y_c \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c \\ z_c + z_s \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c \\ 0 \\ r_c + r_s \end{pmatrix} \right\}$$

$$T = \left\{ \begin{pmatrix} -l \\ x_c \\ y_c + y_s \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ 0 \\ z_c + z_s \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ 0 \\ 0 \\ r_c + r_s \end{pmatrix} \right\}$$

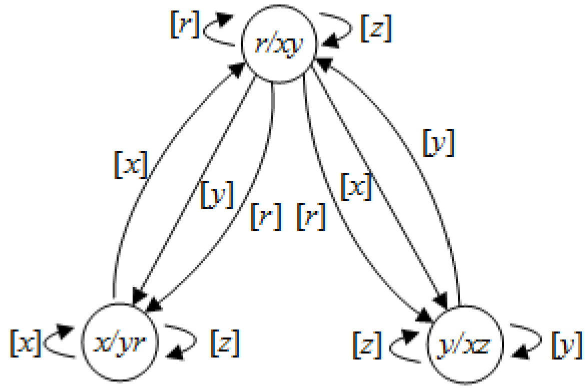

Fig. 4. Partial division of labor.

The complete division of labor and transaction services can be represented by the combination of the following activities vectors: $T = \left\{ \begin{pmatrix} -l \\ x_c+x_s \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c+y_s \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ 0 \\ z_c+z_s \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ 0 \\ 0 \\ r_c+r_s \end{pmatrix} \right\}$.

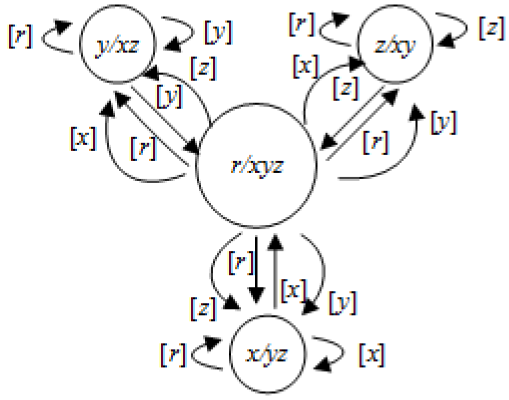

Fig. 5. Complete division of labor and transaction service.

Discussion

In the case of a complete production process, there should be four individuals who produce one type of goods or transactional service.

It is easy to show that the following activity vectors $\left\{ \begin{pmatrix} -l \\ x_c+x_s \\ y_c \\ z_c \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ y_c+y_s \\ z_c \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ y_c \\ z_c+z_s \\ 0 \end{pmatrix} \right\}$ cannot be a part of the production process and a part of labor division because it is impossible to find individuals with the corresponding activity vectors with the positive utility function for these cases.

These four basic forms of the division of labor (autarky, partial division of labor, complete division of labor, and complete division of labor and transaction service) were discussed by X. Yang and Wai-Man Liu [13, p. 115], but the following activities vectors $\left\{ \begin{pmatrix} -l \\ 0 \\ y_c \\ z_c \\ r_c+r_s \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ 0 \\ z_c \\ r_c+r_s \end{pmatrix}, \begin{pmatrix} -l \\ x_c \\ y_c \\ 0 \\ r_c+r_s \end{pmatrix} \right\}$ were not considered.

Activity vector $\begin{pmatrix} -l \\ 0 \\ y_c \\ z_c \\ r_c+r_s \end{pmatrix}$ can exist in the following production process:

$$T_{x_P} = \left\{ \begin{pmatrix} -l \\ 0 \\ y_c \\ z_c \\ r_c + r_s \end{pmatrix}, \begin{pmatrix} -l \\ x_c + x_s \\ 0 \\ z_c \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c + y_s \\ z_c \\ 0 \end{pmatrix} \right\}.$$

The economic structure for this technology is showed in Figure 6. Technology T_{xp} is characterized by the production process with an intermediate product. We can see that *y* is the intermediate product and *x* is the final product because all individuals consume *x* and *y* is used for production *x*.

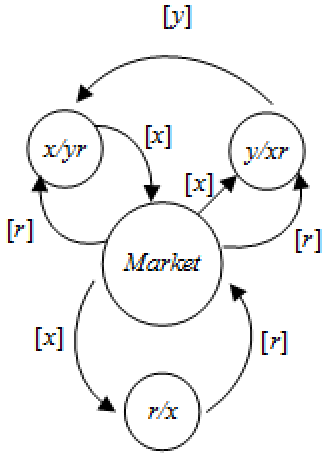

Fig. 6. Division of labor with the intermediate product.

The utility function for configuration *x/yr* can be written as:

$$u_{x/yr} = x_c{}^{\alpha}(r_d y_d)^{\beta} z_c{}^{\gamma}$$

The utility function for configuration *y/xr* can be written as:

$$u_{x/yr} = (r_d x_d)^{\alpha} y_c{}^{\beta} z_c{}^{\gamma}$$

The utility function for configuration *r/x* can be written as:

$$u = (r_c x_d)^{\alpha} y_c{}^{\beta} z_c{}^{\gamma}$$

Another production process with activity vector $\begin{pmatrix} -l \\ 0 \\ y_c \\ z_c \\ r_c + r_s \end{pmatrix}$ can be written the following way:

$$T_{x_F} = \left\{ \begin{pmatrix} -l \\ 0 \\ y_c \\ z_c \\ r_c + r_s \end{pmatrix}, \begin{pmatrix} -l \\ x_c + x_s \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ y_c + y_s \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -l \\ 0 \\ 0 \\ z_c + z_s \\ 0 \end{pmatrix} \right\}.$$

In this case, all individuals decide to specialize in production of final goods. An individual who provided a transactional service makes a decision of partial specialization and purchases final product *x*.

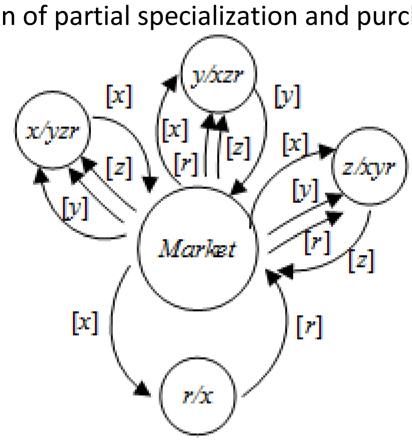

Fig. 7. Division of labor.

This economic structure can exist if the transaction service is different for other types of goods.

Conclusion

Analysis of the technological matrix makes it possible to find all economic structures for a given set of production factors and goods. We can see that all types of predefined economic structures can be found with the help of the technology matrix. We have also considered two economic structures with nonsymmetrical abilities, which were not considered in the initial formulation of the problem.

In the proposed approach, the objective of inframarginal analysis is not just to solve optimization problems for certain economic structures, but to find the economic structures that cannot be determined by experience, and determine their optimality parameters in general equilibrium.

The above-described matrix approach allows us to find and investigate spatially allocated economic structures. We can study the influence of agents who provide logistical support for the decisions that trade partners make in their choice of activities and resource allocation. Modern machine-learning computer methods are applicable for this approach.

References

1. Brecher C., Özdemir D. (2017) Integrative Production Technology Theory and Applications, Springer.
2. Busacker R. G., Saaty T. L. (1973). Finite Graphs and Networks: An Introduction with Applications. New York, McGraw-Hill.
3. Cheng W., Yang X. Inframarginal Analysis of Division of Labor. A Survey. Journal of Economic Behavior & Organization Vol. 55 (2004). 137–174.
4. Cheng W., Zhang D. (2016). How Might the South be Helped by Northern Technology Yet Harmed by Northern Money? Economic Modeling 55, 83–91.
5. Chiang A., Wainwright K. (2005) Fundamental Methods of Mathematical Economics. 4th Edition. New York, McGraw-Hill.
6. Dixon P. (2006) Inframarginal Economics: An Outsider's View. Economic Papers. 25(2), 177–195.
7. Fandel G. (1991) Theory of Production and Cost, Springer-Verlag, Berlin, Heidelberg.
8. Keeney R. L., Raiffa H. (1976) Decisions with Multiple Objectives Preferences and Value Tradeoffs. John Wiley & Sons, Inc.
9. Koopmans T.C. (1951) Analysis of Production as an Efficient Combination of Activities. In: Koopmans T.C., Alchian A, Dantzig G.B., Georgescu-Roegen N, Samuelson PA, Tucker AW (eds) Activity Analysis of Production and Allocation: Conference Proceedings. Wiley, New York, 33–97.
10. Pishchulov G, Richter K. (2016). Optimal Contract Design in the Joint Economic Lot Size Problem with Multi-Dimensional Asymmetric Information. European Journal of Operational Research (2016), 1–23.
11. Wen M. (1998). An Analytical Framework of Consumer-Producers, Economies of Specialization and Transaction Costs. In: Arrow K., Ng, Yew-Kwang, Xiaokai.Y. Increasing Returns and Economic Analysis. UK, Palgrave Macmillan.
12. Tombazos C. (2006). Inframarginal Contributions to Development Economics. Singapore. World Scientific Publishing Co. Pte. Ltd.
13. Yang X., Liu W.-M. (2009). Inframarginal Economics. Increasing Returns and Inframarginal Economics. Vol. 4. Singapore: World Scientific Publishing Co. Pte. Ltd. 904p.
14. Yu X., Chai Y., Liu Y., Sun H. (2014) Infra-Marginal Analysis Model for Provision Mode Selection for E-Commerce Services. Tsinghua Science and Technology, Volume 19, Number 2, April 2014, pp174-183.
15. Zhang Y. (2014). Climate Change and Green Growth: A Perspective of the Division of Labor. China & World Economy / 93–116, Vol. 22, No. 5, 2014.

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